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## Buffer phenomenon in mathematical models of natural sciences

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Speak, that in mathematical model of a nonlinear distributed self-oscillatory system the buffer phenomenon (bufferness) is observed, if in this model there is any predetermined finite numbers of attractors of the same type (stable equilibrium states, stable periodic on time solutions, tori, etc.) for an appropriate choice of its parameters.

On the given problem it is necessary to consider as the first publication the paper of A. A. Vitt [1], the employee and the colleague of A. A. Andronov, his co-author under the classical monograph "Theory of oscillations". A. A. Vitt investigated mathematical model of the auto-generator containing a segment of a long two-wire line with uniformly distributed inductance, capacitance, and resistance; at a heuristic level of rigor he had been specified conditions for existence of the appearance named afterwards by buffer phenomenon. Much later the fact of magnification of number of possible stable self-oscillatory regimes at a variation of parameters of the auto-generator managed to be noticed experimentally [2].

Strict theoretical research of the buffer phenomenon is lead in [3-6] (including system of Vitt [7]) by means of special infinite-dimensional analog of the asymptotic method of Krylov - Bogolyubov - Mitropolskii - Samoilenko.

It has appeared, that the bufferness is typical for a wide class of mathematical models which adequately describe many nonlinear processes in natural sciences (radio-physics [8], mechanics [9], optics [10], theory of combustion [11], ecology [12]). Besides this, connection is traced of buffer phenomenon with such appearances, as origin of turbulence or birth of dynamic chaos [13].

Considered mathematical models represent a boundary value problems for systems of partial differential equations of hyperbolic or parabolic type, and the script of growth of number, for example, stable periodic on time solutions (cycles) is torn as some parameter tends to zero. Essentially, that

the concept “buffer phenomenon” assumes presence a certain bifurcation process as a result of which there is a unlimited growth of number of amount coexisting attractors.

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