
**About Research in Sverdlovsk-Ekaterinburg
Initiated by the Maximum Principle of L.S. Pontryagin**

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In the fifties of the last century in Sverdlovsk a group of young and relatively young graduates from the S.M. Kirov Ural Polytechnical Institute and the A.M. Gorky Ural State University was formed under supervision of two maitres I.G. Malkin and E.A. Barbashin for implementing research in the theory of differential equations and its applications. Professor E. A. Barbashin (1916-1969) – a graduate from the A.M. Gorky USU, was trained in traditions of the Moscow mathematical school in the field of topology, theory of dynamical systems and had achievements recognized in this field. Professor I.G. Malkin (1907-1958) – a graduate from the V. I. Ulyanov-Lenin Kazan State University, was known for his outstanding works in stability theory of motion and nonlinear oscillations in traditions of H. Poincaré and A.M. Lyapunov. In the indicated research group the works were carried out on stability, oscillations and regularization of essentially nonlinear systems, systems with aftereffect, stochastic systems, and works on the theory of abstract dynamical systems.

A new impulse to research was given by publications by L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, and E.F. Mischenko on the mathematical theory of optimal processes [1, 2]. The works by R. Bellman on dynamic programming [3] also affected the research. Later, it was influenced both by the works of R. Kalman on the general theory of dynamical systems [4] and by the monograph of R.A. Isaacs on differential games [5]. In the lecture, it is planned to give a retrospective review of the Sverdlovsk-Ekaterinburg research with the accent on some problem approaches and results which can be related, with more or less grounds, to a class of the “seminal” ones. All this is supposed to be presented in a vivid form with illustration by numerical experiments on the basis of model problems. The stuff of the lecture is generated by research of authors who worked in the Ural: E.G. Albrekht, A.V. Kryazhimskiy, A.B. Kurzhanskii, Yu.S. Osipov, A.I. Subbotin, V.E. Tretyakov, V.N. Ushakov, A.G. Chentsov, S.N. Shimanov. The following topics will be covered. (1) Solution of the time-optimal control problem for a linear system in the problem interpretation as a functional momentum problem. (2) Transformation of the necessary optimality conditions of the maximum principle into sufficient conditions of local optimality for a nonlinear system taking into account the controllability property of

the system of linear approximation. (3) Problems of control and observation. (4) Optimal stabilization of ordinary, hereditary and stochastic systems as a response to the problem of analytic construction of regulators ascending to A.M. Letov [6]. (5) A conflict pursuit-evasion problem with transition to the general concept of a positional differential game. (6) The pursuit-evasion alternative on the basis of the family of “constructive” motions. (7) Unification of a differential game. (8) A generalized minimax solution of a differential equation of the Hamilton-Jacobi type and its development not only for ordinary dynamical systems.

P.S. The author also intends to share memories about his talks with L.S. Pontryagin on questions related to subjects discussed in the lecture.

References

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