Control Bifurcations

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A parametrized nonlinear differential equation can have multiple equilibria as the parameter is varied. A local bifurcation of a parametrized differential equation occurs at an equilibrium where there is a change in the topological character of the nearby solution curves. This typically happens because some eigenvalues of the parametrized linear approximating differential equation cross the imaginary axis and there is a change in stability of the equilibrium. The topological nature of the solutions is unchanged by smooth changes of state coordinates so these may be used to bring the differential equation into Poincaré normal form. From this normal form, the type of the bifurcation can be determined. For differential equations depending on a single parameter, the typical ways that the system can bifurcate are fully understood, e.g., the fold (or saddle node), the transcritical and the Hopf bifurcation.

A nonlinear control system has multiple equilibria typically parametrized by the set value of the control. A control bifurcation of a nonlinear system typically occurs when its linear approximation loses stabilizability. The ways in which this can happen are understood through the appropriate normal forms. We present the quadratic and cubic normal forms of a scalar input nonlinear control system around an equilibrium point. These are the normal forms under quadratic and cubic change of state coordinates and invertible state feedback . The system need not be linearly controllable. We study some important control bifurcations, the analogues of the classical fold, transcritical and Hopf bifurcations.

References

[1] A. J. Krener, W. Kang, and D. E. Chang, "Control Bifurcations", *IEEE Trans. Automatic Control*, 49, 1231–1246 (2004).