Toric Orbifolds

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From the viewpoint of algebraic geometry, a toric variety X is the compactification of an algebraic torus $(\mathbb{C}\setminus 0)^n$ using the combinatorial instructions provided by a *fan* in $\mathbb{R}^n$; the procedure is such that coordinatewise multiplication on the torus extends to an action on X. If X is locally homeomorphic to the quotient of $\mathbb{C}^n$ by the action of a finite group, then it is a *toric orbifold.* A particularly interesting family of examples are given by the *weighted projective spaces* $\mathbb{C}P^n(\chi)$, which depend upon a vector of positive integral weights $(\chi_0, \ldots, \chi_n)$ for their definition. In recent years, these spaces have made an appearance in several areas of mathematics, including algebraic and symplectic geometry, and theoretical physics.

The primary aim of this talk is to present the philosophy of toric topology in the context of toric orbifolds, with regular references to the $\mathbb{C}P^n(\chi)$. I shall attempt to make the talk as accessible as possible to a general audience, and many details will be omitted. Nevertheless, the ultimate goal is to outline recent work with Tony Bahri and Matthias Franz, in which we compute the equivariant integral cohomology ring $H^*_T(\mathbb{C}P^n(\chi))$, with respect to the action of the standard compact n-torus $T < (\mathbb{C}\setminus 0)^n$. The conclusions are highly dependent on the number theoretic relations amongst the weights χ_j , as might be expected.

Our calculations depend on a result of Franz and Puppe concerning the exactness of the Chang-Skjelbred cohomology sequence with integral coefficients. They are expressed in terms of the algebra of *piecewise polynomials* associated to the fan, and may be considered to be purely combinatorial. However, I shall also refer to the more inclusive viewpoint that we are currently developing, which relates the calculations to weighted lens spaces, homotopy colimits, the Bousfield Kan spectral sequence, and weighted face rings. As time permits I shall describe the most important general features of each of these inputs, which lie close to the heart of toric topology.

References

[1] Tony Bahri, Matthias Franz, and Nigel Ray, "The equivariant cohomology of weighted projective spaces" *arXiv*, 11, No. 22, 333444 (2007).